Optimization of Focal Length for a Parabolic Solar Cooker Design

Hartmut Ehmler, last modified on 24 September 2013

Abstract

A study of parabolic solar cooker design is carried out in order to develop a simple model for the variation of cooking power with imperfect adjustment in the horizontal plane. It turns out that the main parameters to be considered are the focal length and the cooking pot diameter. For a given arc length of the reflector there is an optimum geometry where the energy input to the cooking pot can be maximized. A parameter study is performed for the Lightoven solar cooker type. This enables us to choose the focal length in the design of a new model.

Motivation

Parabolic solar cooker designs are quite common. The reflector has to track the sun in order to maintain the maximum cooking power. How often the cooker has to be adjusted depends on the solar path during cooking and the cooker design. Some parabolic designs are quite sensitive to imperfect adjustments whereas others are not. The aim of the present study is to develop a simple model for the variation of cooking power with imperfect adjustment in the horizontal plane and to optimize the cooker design accordingly.

Model

Figure 1 shows the basic geometry of a parabolic cooker in two dimensions. This applies to both - a paraboloid or a parabolic trough reflector. All parameters which are relevant for the following calculations are indicated in this figure.

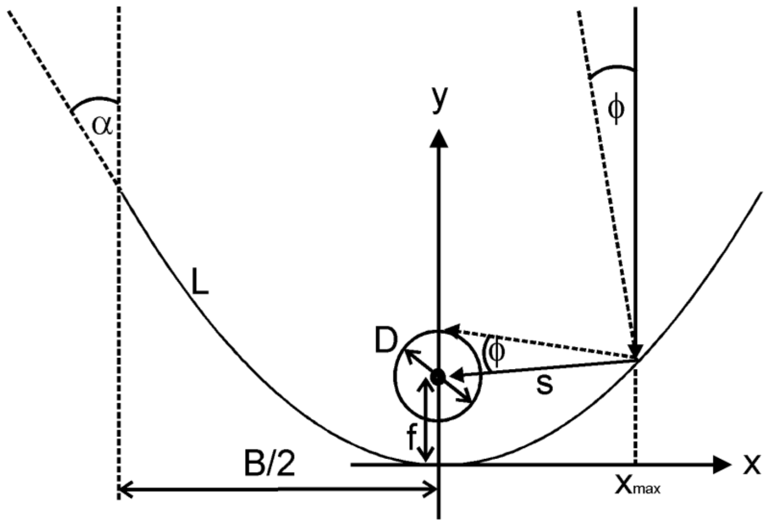

Figure 1: Basic geometry of a parabolic solar cooker where the incident rays have a certain deviation angle with respect to ideal adjustment to the sun.

An incident ray is hitting the reflector at a point (x,y). Let the ray have an angle φ with respect to the ideal direction. The reflected ray will just hit the cooking vessel at the edge under the condition:

$$\varphi = \sin^{-1}\{D/(2s)\}, \qquad (1)$$

where D is the diameter of the cooking vessel and s the distance from the focal point to the hitting point (x,y) on the reflector.

For the distance s follows from vector addition:

$$s = \sqrt{\frac{1}{2}x^2 + \left(\frac{x^2}{4f}\right)^2 + f^2}. \qquad (2)$$

By formula (1) and (2) it can be determined for each position x on the reflector how large the maximum deviation angle can be. In other words, for a given deviation angle φ there is a position x_{max} on the reflector where the rays with x< x_{max} will hit the reflector and rays with x> x_{max} will not. The cooking power P will vary as the ratio $x_{max}/(B/2)$ because the effective width (aperture) is reduced by this factor:

$$P \;\; \varphi \;\; = P_0 \frac{x_{\max}(\varphi)}{B/2} \times cos\varphi \, . \qquad (3)$$

The factor $cos\varphi$ takes into account the reduced aperture due to the deviation angle itself.

The cooking power P_0 for ideal adjustment ($\varphi = 0$) is proportional to the aperture. The aperture is proportional to the width B (or diameter) of the reflector.

$$P_0 \propto B \, . \qquad (4)$$

The width of the reflector depends on the focal distance and the arc length L of the reflector through

$$L = \frac{1}{4a} \ln(2ax + \sqrt{1 + 4a^2x^2}) + \frac{1}{2}x\sqrt{1 + 4a^2x^2} + c \qquad (5)$$

where the parabola is given by $y=ax^2$ and the focal distance is

$$f = \frac{1}{4a} \qquad (6)$$

which allows to calculate the width B for a set of different focal lengths and a given total arc length. The opening aperture decreases with decreasing focal length since the parabola is “closing” more and more.

If the deviation angle is too large the reflector will partially shadow itself. This will start at the reflector edges where the opening angle α is given by the slope of parabola (see Figure 1):

$$\alpha = \cot^{-1}\left(\frac{dy}{dx}\right)_{y=B/2} = \cot^{-1}\left(\frac{B}{4f}\right). \qquad (7)$$

All calculations will be limited to deviation angles smaller than the opening angle, i.e. $\varphi < \alpha$ because otherwise partial shadowing occurs. The model assumes that the cooking power drops to zero as soon as the reflector starts to shadow itself from its edges. This is a simplification because in reality for a deviation angle $\varphi > \alpha$ there is still a finite albeit small cooking power remaining until complete shadowing occurs – provided that in a cooker with a long focal length this deviation angle is not too large for all rays being able to reach the cooking vessel.

The total energy E while the sun is crossing the acceptance area of the reflector can be found by integration over the opening angle:

$$E = \int_{-\alpha}^{+\alpha} P(\varphi) d\varphi \, . \qquad (8)$$

Method

Excel is used as a calculation tool. The calculations proceed along the following steps:

a. Calculate the width B of the reflector for the given total arc length (L=120 cm for Lightoven production reasons) and a set of different focal distances from 50 cm down to 5 cm (Eq. 5,6)
b. Calculate for this set the cooking power P_0 and normalize it to 100% for infinite focal distance (Eq. 4)
c. Determine the opening angle α for each geometry
d. Calculate for each point (x,y) the distance S and the corresponding deviation angle φ (Eq. 1-2)
e. Take this set of data points and derive the relative cooking power $P \;\; \varphi$ from Eq. 3
f. Integrate the curve $P \;\; \varphi$ over the whole range of angles smaller than the opening angle α (Eq. 8)
g. Repeat these calculations for different pot diameters D and compare the results

Results

Cooking power

First the cooking power is considered for an ideal orientation towards the sun (φ =0). The focal distance is considered as a free parameter while the total arc length of the reflector is fixed. The cooking power is normalized to 100% for an infinite focal length where the parabola would be completely flat. The smaller the focal length the more decreases the aperture and so the maximum available cooking power. This is shown in Figure 2.

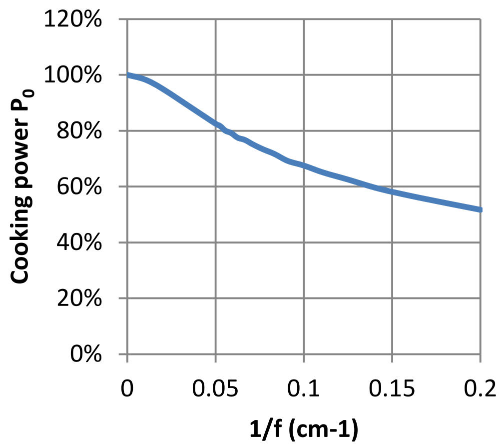

Figure 2: Cooking power of a parabolic reflector with perfect adjustment to the sun. The decrease is due to aperture for decreasing focal length f. Numbers are calculated for the Lightoven I+II reflector with an arc length of 120 cm.

Next the cooking power variation is studied for a deviation angle φ >0, i.e. non-ideal adjustment due to the sun movement in the horizontal plane during cooking. Figure 3 shows the results for the Lightoven I. For a horizontal deviation of smaller than 5° in both directions all rays on the reflector can hit the cooking vessel[1]. The cooking power decreases to 50% of the maximum value after an angle of about 8° which takes about 20 minutes during noon-time in summer.

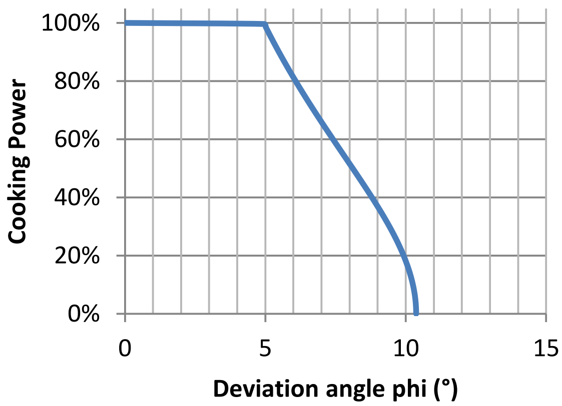

Figure 3: Relative variation of cooking power with azimuthal deviation of the sun angle with respect to ideal adjustment calculated for the geometry of the Lightoven I.

[1] This neglects imperfections in the reflector which would smear out this curve in practice. The actual curve could be measured by the reciprocal optical method developed by Bill Bradley (http://www.earthboundtech.com/cookerperformanceeval.htm) .

Cooking energy

Now the total energy is considered when the solar cooker is not adjusted[2] and the sun covers the full range of the acceptance angle. This means basically integration over the full angle range of Figure 3 according to Eq. (8). The cooking process will follow the energy input corresponding to the deviation angle. When decreasing the focal length to effects counteract: The maximum available cooking power decreases with focal length but the integral power over the angle range of Figure 3 increases until a certain optimum. This contribution comes from longer "wings" of the curve in Figure 3 in where a significant portion of the rays can reach the cooking vessel in low distance from the reflector. Figure 4 shows the results for different focal length and pot diameter.

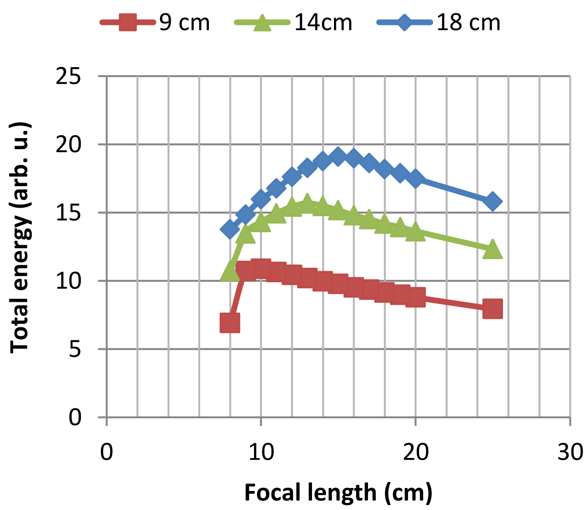

Figure 4: Total energy input to the cooking vessel if the reflector is not adjusted and the sun path covers the full range of acceptance angle. Parameters are the same as in Figure 2. The cooking vessel diameter is varied as parameter (see legend).

One can see in Figure 4 that there is a maximum in the energy input for a medium focal length which is however quite broad. It shifts to higher focal lengths with increasing cooking pot diameter.

Conclusion

Cookers which are optimized for maximum power at perfect adjustment will use a focal length as high as practically achievable. These cookers will require frequent tracking. On the other hand if the cooker is intended for cooking in absence the total energy during the whole cooking process should be maximized although the cooker is only temporarily adjusted. This leads to a design with a smaller focal length. This strategy will be followed for a new model of the Lightoven which is meant to cook food with as few adjustments (if any) to the solar path as possible.

---

[2] In practice this corresponds to the situation where the cooker is set for a position where the sun will be at a later time. This is the case how box cookers can cook food in absence.