

Mathematics (Compulsory Part) – Summary on techniques on counting and probability

A. **Basic techniques** (addition, subtraction, multiplication, division)

B. **Advanced (general) techniques**

Factorial, permutation, combination

Recall: $P_r^n =$, $C_r^n =$

Useful results: $P_1^n =$, $P_n^n =$, $P_r^n P_{k-r}^{n-r} =$

$C_1^n =$ (=), $C_2^n =$, $C_r^n C_{k-r}^{n-r} =$

C. **Advanced (special) techniques**

<p>The number of ways to arrange 10 people in a queue if</p> <p>(i) there is no restriction,</p> <p>(ii) 3 of them must be in 3 specific positions,</p> <p>(iii) 3 of them must be together,</p> <p>(iv) 3 of them must be separated.</p>	<p>The number of ways to arrange n people in a queue if</p> <p>(i) there is no restriction,</p> <p>(ii) r of them must be in r specific positions,</p> <p>(iii) r of them must be together,</p> <p>(iv) r of them must be separated.</p>
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Note: People are not together DOES NOT mean they are all separated (except when there are TWO persons).

When there are three boys and two girls, how many ways are to arrange them if

(i) there is no restriction,

(ii) the two girls are together,

(iii) the two girls are separated.

The number of ways to choose 5 people from 4 girls and 6 boys if

- (i) there is no restriction,
- (ii) exactly 2 girls are chosen,
- (iii) at least 2 girls are chosen,
- (iv) at least 1 girl and 2 boys are chosen.

Four persons are chosen from 5 couples. Find the number of ways that the four persons consist of

- (i) two couples,
- (ii) exactly one couple,
- (iii) no couple,
- (iv) at least one couple.

[Difficult!] There are 5 groups and there are three persons in each group. Find the number of ways to choose five persons if

- (i) they come from five different groups,
- (ii) all persons from a certain group are chosen,
- (iii) all persons from any group are chosen,
- (iv) they come from two different groups,
- (v) they come from three different groups.

D. Probability

There are 2 red balls and 3 green balls in bag A, while there are 2 red balls and 2 green balls in bag B.

(a) A ball is randomly taken from bag A and then put it in bag B. Afterwards, a ball is randomly taken from bag B and then put it in bag A.

(i) Find the probability that there is no change in the number of balls of each colour in each bag finally.

(ii) Given that there is no change in the number of balls of each colour in each bag finally, find the probability that the ball taken from bag A is red.

(b) A ball is randomly taken from bag A and then put it in bag B. Then a ball is chosen from each bag.

(i) If the ball taken from bag A is red, find the probability that the two balls are of the same colour.

(ii) If the two balls are of the same colour, find the probability that the ball taken from bag A is red.

Form 5 Mathematics – Exercises on permutation and combination

1. Find the number of ways to choose seven students from a Form 5 class (with 6 boys and 18 girls) lining up in a queue if
 - (a) there is no restriction,
 - (b) Caleb must not be chosen,
 - (c) Caleb must be chosen and in the last position,
 - (d) Alex, Mike and Amman must be chosen and in the first three positions,
 - (e) there are 3 boys and then 4 girls,
 - (f) there are 2 girls, 3 boys, and then 2 girls,
 - (g) there is at least one girl.

For Q.2 and Q.3, there are seven selected students from a Form 5 class:

Alex, Eric, Claudia, Anna, Alexandra, Joanne, Carrie.

2. Find the number of ways for them to line up in a queue if
 - (a) there is no restriction,
 - (b) Eric must be in the first position,
 - (c) all boys are in front of girls,
 - (d) all girls are in front of boys,
 - (e) all boys are in front or all girls are in front,
 - (f) there are 2 girls first, then all boys, and then the rest of girls,
 - (g) all boys must be together,
 - (h) all girls must be together,
 - (i) all boys or all girls must be together,
 - (j) Claudia and Carrie cannot be together,
 - (k) Alex, Anna and Alexandra cannot be together,
 - (l) no two girls are together.

3. There are two rows of seats, the first row with 2 seats and the second row with 5 seats. Find the number of ways for them to be seated if
 - (a) there is no restriction,
 - (b) the girls are all in one row,
 - (c) Claudia must be in the first row,
 - (d) Eric and Carrie must be in the second row,
 - (e) Anna and Joanne must be together,
 - (f) Anna, Alexandra, Joanne and Carrie must be separated.